

# Short-range tensor interaction and high-density nuclear symmetry energy

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Effects of the short-range tensor interaction on the density-dependence of nuclear symmetry energy are examined by applying an approximate expression for the second-order tensor contribution to the symmetry energy derived earlier by G.E. Brown and R. Machleidt. It is found that the uncertainty in the short-range tensor force leads directly to a divergent high-density behavior of the nuclear symmetry energy.

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The density dependence of nuclear symmetry energy  $E_{\text{sym}}(\rho)$  encodes the energy related to neutron-proton asymmetry in the Equation of State (EOS) of nuclear matter. While the  $E_{\text{sym}}(\rho)$  is very important for both nuclear physics and astrophysics [1–14], it is still rather uncertain especially at supra-saturation densities. Besides promising constraints being extracted from astrophysical observations [16], significant progress has been made recently in constraining the  $E_{\text{sym}}(\rho)$  around and below the nuclear matter saturation density  $\rho_0$  using experiments in terrestrial laboratories [17]. Looking forward, it is very exciting to note that dedicated experiments are currently underway or being planned at several advanced radioactive ion beam facilities at CSR/China [15], FRIB/USA [18], GSI/Germany [19], RIKEN/Japan [20] and KoRIA/Korea [21] to pin down the high-density behavior of the  $E_{\text{sym}}(\rho)$ . While essentially all existing many-body theories have been used to predict the  $E_{\text{sym}}(\rho)$ , the results diverge quite widely especially at supra-saturation densities, see, e.g., ref. [5] for a recent review. Thus, it is necessary to identify fundamental reasons for the uncertain high-density behavior of the  $E_{\text{sym}}(\rho)$ . Generally speaking, besides the different techniques often used in treating nuclear many-body problems in various theories, our poor knowledge about the isospin dependence of the in-medium nuclear strong interaction is at least partially responsible for the uncertain  $E_{\text{sym}}(\rho)$ . In fact, it has been recognized that the spin-isospin dependence of the three-body force, see, e.g., ref. [9, 22], the isospin dependence of short-range nucleon-nucleon correlation functions, see, e.g., ref. [23], and the short-range tensor force, see, e.g., ref. [24] all play some significant roles in determining the high-density behavior of the  $E_{\text{sym}}(\rho)$ . In particular, it is easy to understand qualitatively why the nuclear tensor interaction is important in determining the  $E_{\text{sym}}(\rho)$ . Within the parabolic approximation of the EOS of isospin asymmetric nuclear matter, see, e.g., ref. [25], the  $E_{\text{sym}}(\rho)$  can be written as the difference between the nucleon specific energy in pure neutron matter (PNM) and symmetric nuclear matter (SNM), i.e.,  $E_{\text{sym}}(\rho) = E_{\text{PNM}}(\rho) - E_{\text{SNM}}(\rho)$ . It is well known that in the isospin-singlet  $T = 0$  nucleon-nucleon interaction channel relevant for calculating the EOS of SNM, a

significant tensor component is required to understand properties of deuteron and neutron-proton scattering data, see, e.g., refs. [26, 27]. Moreover, it has been found consistently in microscopic many-body calculations that the  $T = 0$  channel dominates the potential contribution to the symmetry energy [25, 28]. In this note, using several typical and widely used tensor forces that are the same at long-range but have characteristically different short-range behaviors, we examine effects of the short-range tensor force on the  $E_{\text{sym}}(\rho)$ . Applying an approximate expression for the second-order tensor contribution to the symmetry energy derived earlier by G.E. Brown and R. Machleidt [39], we find that the uncertainty in the short-range tensor force contributes significantly to the divergence of the  $E_{\text{sym}}(\rho)$  at supra-saturation densities.

In the best-studied phenomenology of nuclear forces, i.e., the one-boson-exchange model, the tensor interaction results from exchanges of the isovector  $\pi$  and  $\rho$  mesons. For instance, the tensor part of the one-pion exchange potential (OPEP) can be written in configuration space as [26]

$$V_{\pi} = -\frac{f_{\pi}^2}{4\pi} m_{\pi} (\tau_1 \cdot \tau_2) S_{12} \left[ \frac{1}{(m_{\pi} r)^3} + \frac{1}{(m_{\pi} r)^2} + \frac{1}{3m_{\pi} r} \right] \exp(-m_{\pi} r) \quad (1)$$

where  $r$  is the inter-particle distance and  $S_{12} = 3 \frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - (\sigma_1 \cdot \sigma_2)$  is the tensor operator. The  $\rho$ -exchange tensor interaction  $V_{\rho}$  has the same functional form as the OPEP, but with the  $m_{\pi}$  replaced everywhere by  $m_{\rho}$ , and the  $f_{\pi}^2$  by  $-f_{\rho}^2$ . The magnitudes of both the  $\pi$  and  $\rho$  contributions grow quickly with decreasing  $r$ . A proper cancelation of the opposite contributions from the  $\pi$  and  $\rho$  exchanges is supposed to give a realistic strength for the nuclear tensor force. However, since the tensor coupling is not well determined consistently from deuteron properties and/or nucleon-nucleon scattering data, the tensor interaction is by far the most uncertain part of the nucleon-nucleon interaction [27]. Moreover, it is also possible that the in-medium  $\rho$  meson mass  $m_{\rho}$  is different from its free-space value [29]. A density-dependent in-medium  $m_{\rho}$  will lead to very different short-range tensor force [30] and affects the symmetry energy at high densities [9, 31, 32]. While there is no community-wide consensus on whether the  $m_{\rho}$  changes or not in the dense medium, it is a possible origin for the uncertain short-range tensor force. In addition, due to both the physical and mathematical differences in construction [27],

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various realistic nuclear potentials usually have widely different tensor components at short range ( $r \leq 0.8$  fm). For example, in the Paris potential [33], it is just described simply by a constant soft core. The Argonne V18 (Av18) uses local functions of Woods-Saxon type [34], while Reid93 applies local Yukawas of multiples of the pion mass [35]. While it is promising that new experiments, such as, (p,d) reactions induced by high energy protons [36] or two nucleon knockout reactions induced by high energy electrons [37, 38], may allow us to better constrain the short-range tensor force in the near future, currently the short-range behavior of the tensor force is still very uncertain.

It is easy to see from Eq. (1) that the expectation value of the tensor force  $\langle V_t \rangle$  is zero. Thus, the first-order tensor force does not contribute to the symmetry energy unless one assumes that all isosinglet neutron-proton pairs behave as bound deuterons with  $S_{12} = 2$  [9]. In fact, it is the second-order tensor contribution that is important for the binding energy of nuclear matter [40, 41] and thus also for the symmetry energy [39]. Using a second-order effective tensor interaction obtained first by Kuo and Brown [40], see. e.g., ref. [42] for a review, Brown and Machleidt found that the tensor contribution to the symmetry energy is approximately

$$\langle V_{sym} \rangle = \frac{12}{e_{\text{eff}}} \langle V_t^2(r) \rangle \quad (2)$$

where  $e_{\text{eff}} \approx 200$  MeV and  $V_t(r)$  is the radial part of the tensor force [39]. While this approximate expression may lead to symmetry energies systematically different from predictions of advanced microscopic many-body theories using various interactions, it is handy to evaluate effects of the different short-range tensor forces within the same simple and analytical approach. Of course, it is necessary and also interesting to evaluate the accuracy of Eq. (2) with respect to microscopic many-body calculations using the same interaction.

To apply Eq. (2) we evaluate the expectation value of  $V_{\text{sym}}$  using the free single-particle wave function ( $V^{-1} e^{i\mathbf{k}\cdot\mathbf{r}}$ )  $\eta_\lambda \zeta_\tau$ , where  $\eta_{\lambda=\uparrow/\downarrow}$  and  $\zeta_{\tau=p/n}$  is the spin and isospin wave function, respectively. The direct and exchange matrixes are, respectively,

$$\begin{aligned} & \langle \mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau' | V_{\text{sym}} | \mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau' \rangle \\ &= \frac{1}{V^2} \int d^3r \int d^3r' e^{-i\mathbf{k}\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} \eta_\lambda^\dagger(1) \eta_{\lambda'}^\dagger(2) \zeta_\tau^\dagger(1) \zeta_{\tau'}^\dagger(2) \\ & \quad \times V_{\text{sym}}(1,2) e^{i\mathbf{k}\cdot\mathbf{r}} e^{i\mathbf{k}'\cdot\mathbf{r}'} \eta_\lambda(1) \eta_{\lambda'}(2) \zeta_\tau(1) \zeta_{\tau'}(2) \\ &= \frac{1}{V} \int V_{\text{sym}}(\mathbf{r}) d^3r \end{aligned} \quad (3)$$

and

$$\begin{aligned} & \langle \mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau' | V_{\text{sym}} | \mathbf{k}'\lambda' \tau' \mathbf{k}\lambda \tau \rangle \\ &= \frac{1}{V^2} \int d^3r \int d^3r' e^{-i\mathbf{k}\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} \eta_\lambda^\dagger(1) \eta_{\lambda'}^\dagger(2) \zeta_\tau^\dagger(1) \zeta_{\tau'}^\dagger(2) \\ & \quad \times V_{\text{sym}}(1,2) e^{i\mathbf{k}'\cdot\mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}'} \eta_{\lambda'}(1) \eta_\lambda(2) \zeta_{\tau'}(1) \zeta_\tau(2) \\ &= \frac{1}{V} \delta_{\lambda\lambda'} \delta_{\tau\tau'} \int \exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}] V_{\text{sym}}(\mathbf{r}) d^3r. \end{aligned} \quad (4)$$

The expectation value of  $V_{\text{sym}}$  in the  $S = 1, T = 0$  channel is

thus

$$\begin{aligned} & \langle V_{\text{sym}} \rangle \\ &= \frac{1}{16} \frac{1}{2} \sum_{\mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau'} [(\mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau' | V_{\text{sym}} | \mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau') \\ & \quad - (\mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau' | V_{\text{sym}} | \mathbf{k}'\lambda' \tau' \mathbf{k}\lambda \tau)] \\ &= \frac{1}{32} \sum_{\mathbf{k}\lambda \tau \mathbf{k}'\lambda' \tau'} \frac{1}{V} \left\{ \int V_{\text{sym}}(\mathbf{r}) d^3r \right. \\ & \quad \left. - \delta_{\tau\tau'} \delta_{\lambda\lambda'} \int \exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}] V_{\text{sym}}(\mathbf{r}) d^3r \right\} \\ &= \frac{V}{2} \frac{1}{(2\pi)^6} \int^{k_F} d^3k \int^{k_F} d^3k' \left\{ \int V_{\text{sym}}(\mathbf{r}) d^3r \right. \\ & \quad \left. - \frac{1}{4} \int \exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}] V_{\text{sym}}(\mathbf{r}) d^3r \right\}. \end{aligned} \quad (5)$$

Noticing that the momentum integral

$$\begin{aligned} \int^{k_F} d^3k e^{i\mathbf{k}\cdot\mathbf{r}} &= 4\pi \int_0^{k_F} k^2 j_0(kr) dk \\ &= \frac{4\pi k_F^3}{3} \frac{3j_1(k_F r)}{k_F r} \end{aligned} \quad (6)$$

and the particle number density  $\frac{A}{V} = \frac{2}{3\pi^2} k_F^3$ , we can write the tensor contribution to the symmetry energy as

$$\begin{aligned} \frac{\langle V_{\text{sym}} \rangle}{A} &= \frac{12}{e_{\text{eff}}} \cdot \frac{k_F^3}{12\pi^2} \left\{ \frac{1}{4} \int V_t^2(r) d^3r \right. \\ & \quad \left. - \frac{1}{16} \int \left[ \frac{3j_1(k_F r)}{k_F r} \right]^2 V_t^2(r) d^3r \right\}. \end{aligned} \quad (7)$$

For large  $k_F$ , the second integral in the above equation approaches zero, the first term is thus expected to dominate at high densities, leading to an almost linear density dependence.

To access quantitatively effects of the short-range tensor force on the density dependence of nuclear symmetry energy, we adopt here several tensor forces used by Otsuka et al. in their recent studies of nuclear structures [43]. The considered tensor forces, including the standard  $\pi + \rho$  exchange (labelled as *a*), the G-Matrix (GM) [43] (labelled as *b*), M3Y [44] (labelled as *c*) and the Av18 [34] (labelled as Av18), as shown in the left panel of Fig. 1, behave rather differently at short distance, but merge to the same Av18 tensor force at longer range. In addition, we add a case (*d*) where the tensor force vanishes for  $r \leq 0.7$  fm. The  $\pi + \rho$  exchange interaction is fixed by the standard meson-nucleon coupling constants with a strong  $\rho$  coupling [42], and we use a short-range cut-off at  $r = 0.4$  fm, i.e.,  $V(r < 0.4 \text{ fm}) = V(r = 0.4 \text{ fm})$ . As emphasized by Otsuka et al. [43], the short-range behavior of the tensor force has no effect on nuclear structures. However, as we shall show in the following, it affects significantly the  $E_{\text{sym}}(\rho)$  especially at supra-saturation densities.

Shown in the right panel of Fig. 1 are the potential parts of the symmetry energies due to the tensor forces considered according to Eq. (7). As expected, they tend to grow linearly with increasing density. Since it is the square of the tensor force that determines its contribution to the symmetry energy,

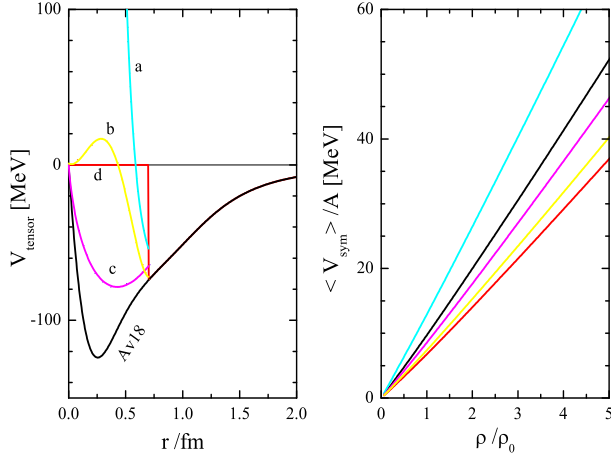


FIG. 1: (Color online) Left panel: radial parts of the tensor interactions having different short-range behaviors but the same long-range ( $r > 0.7\text{fm}$ ) part as the Av18, Right panel: potential part of the symmetry energy with the different short-range tensor interactions.

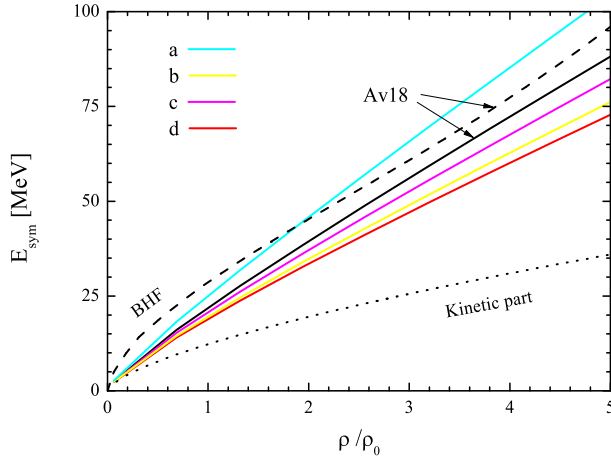


FIG. 2: (Color online) Symmetry energies using various short-range tensor interactions in Eq. (2) in comparison with the Brueckner-Hartree-Fock prediction using the Av18 potential.

tensor forces having larger magnitudes at short distance affect more significantly the symmetry energy. It is seen that the variation of the tensor force at short distance affects significantly the high-density behavior of nuclear symmetry energy. Including also the kinetic part of the symmetry energy  $\frac{1}{3} \frac{k_F^2}{2m}$ , we show in Fig. 2 the  $E_{\text{sym}}(\rho)$ . The divergent values of the  $E_{\text{sym}}(\rho)$  are completely due to the different short-range tensor

forces used. To evaluate the accuracy of the results obtained using Eq. (2), we compare in Fig. 2 predictions from Eq. (2) and the Brueckner-Hartree-Fock (BHF) [22] both using the Av18 interaction. It is seen that essentially over the whole density range considered, the BHF prediction is about 7 MeV higher. This is qualitatively understandable since the difference in central forces between the isotriplet  $T = 1$  and isosinglet  $T = 0$  channels also contribute to the potential part of the symmetry energy [24, 45]. The comparison here indicates clearly that indeed, as expected by G.E. Brown and R. Machleidt [39], the tensor contribution dominates the potential part of the nuclear symmetry energy. The 7 MeV difference can be considered as the systematic error of predictions based on Eq. (2). Thus, it is clear that the variation of the short-range tensor force leads to significantly different symmetry energies at supra-saturation densities. To be accurate, nevertheless, one should be cautioned that the uncertain short-range tensor force is probably not the only reason for the poorly known high-density behavior of the  $E_{\text{sym}}(\rho)$ . There are also correlations among probably several factors that may all affect the  $E_{\text{sym}}(\rho)$  individually. For example, the short-range tensor force also leads to neutron-proton correlations in SNM [46]. Consequently, the single-nucleon momentum distribution obtains a high momentum tail that will change the average kinetic energy of nucleons in SNM [47], and thus the kinetic part of the  $E_{\text{sym}}(\rho)$  [23]. While this effect is not considered here, our results based on Eq. (2) are interesting and useful for better understanding the role of tensor forces in determining the  $E_{\text{sym}}(\rho)$ .

In summary, using an approximate expression for the second-order tensor contribution to the symmetry energy derived earlier by G.E. Brown and R. Machleidt, we investigated effects of the short-range tensor interaction on the density-dependence of nuclear symmetry energy. We found that indeed the tensor force dominates the potential part of the nuclear symmetry energy. The uncertain short-range tensor force contributes significantly to the divergence of the nuclear symmetry energy especially at supra-saturation densities.

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